Assimilation of MIPAS data: impact of nonlinearity

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BACKGROUND

Assimilation of remote sounder data has not done well in the past.

- *Assimilating retrievals* has problems:
  - ➢finite vertical resolution,
  - ➢non-diagonal covariance,
  - ➢influence of a priori.

Inadequate treatment has meant that much of the information content has been ignored.

- *Assimilating radiances* is better, but still has problems:
  - ➢complicated and time consuming forward models have to be approximated
  - ➢sometimes measurement errors are not independent
  - ➢systematic errors are usually not independent
  - ➢modern high spectral resolution instruments have a high data rate

An Alternative Approach

Assimilation with a view to information content:

- ➢represent the data in a form that is linear in assimilated parameters, with errors which are independent
- ➢the linearisation must be valid over an appropriate range of the parameters
- ➢provide a full characterisation of the transformed data

Possibilities are:

- ➢linearised and prewhitened forward model, evaluated at an offline retrieval
- ➢averaging kernel representation of the retrieval, prewhitened
- ➢either of these can be compressed by the use of singular vectors of the linear model

Critical insight:

The linearisation need only be valid within the error bounds of the retrieval. This applies to both approaches.

Example: showing how a priori is removed, using the averaging kernels

The retrieval vector $\mathbf{x}_r$ can be related to the true state $\mathbf{x}$ by an equation of the form

$$\mathbf{x}_r = \mathbf{x}_a + \mathbf{A}(\mathbf{x} - \mathbf{x}_a) + \varepsilon$$

where ε is experimental error with covariance $\mathbf{S}_\varepsilon$, and $\mathbf{x}_a$ is the a priori and $\mathbf{A}$ is the averaging kernel matrix.

We can rearrange this as:

$$\mathbf{y} = \mathbf{x}_r - \mathbf{x}_a + \mathbf{A}\mathbf{x}_a = \mathbf{A}\mathbf{x} + \varepsilon$$

so that $\mathbf{y}$ is an estimate of $\mathbf{Ax}$ with error $\boldsymbol{\varepsilon}$. Premultiplication of both sides by a square root of $\mathbf{S}_\varepsilon$, such as its Cholesky decomposition, will transform the error to be a unit matrix.

The purpose of this paper is to determine whether MIPAS products can be assimilated in this way.

Therefore we investigate the linearity of the MIPAS forward model within the error bounds of real retrieved data.

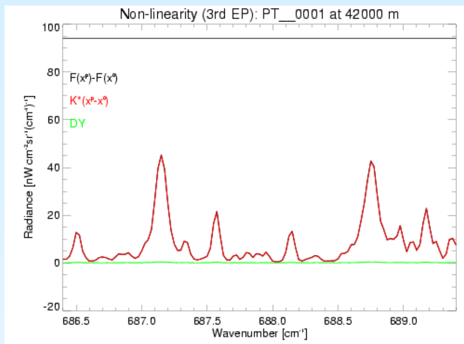
Degree of Nonlinearity

For any particular problem the degree of nonlinearity can be examined by evaluating:

$$\delta\mathbf{y} = [\mathbf{F}(\mathbf{x}_r) - \mathbf{F}(\mathbf{x}) - \mathbf{K}(\mathbf{x}_r - \mathbf{x})]$$

for values of $\mathbf{x}$ at the equivalent of one standard deviation away from $\mathbf{x}_r$ by using $\mathbf{x} = \mathbf{x}_r \pm e_i$ where e_i is an error pattern of the solution error covariance $\mathbf{S}_r$. The size of $\delta\mathbf{y}$ can be evaluated by comparison with $\mathbf{S}_\varepsilon$ using $c^2 = \delta\mathbf{y}^T\mathbf{S}_\varepsilon^{-1}\delta\mathbf{y}$, a quantity like a χ^2.

Figure 1: Example of the comparison between the forward model (**F**) and the linearised forward model (**K·x**) for a pT microwindow.



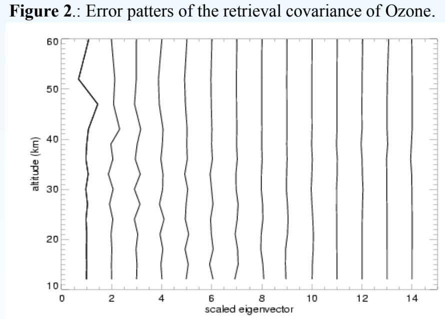
Error Patterns

Basis in which the errors are independent by diagonalising the covariance matrix, $\mathbf{SL} = \mathbf{L}\Lambda$ (for a symmetric matrix: $\mathbf{L}^{-1} = \mathbf{L}^T$)

$$\mathbf{S} = \Sigma_i \lambda_i \mathbf{l}_i \mathbf{l}_i^T = \Sigma_i \mathbf{e}_i \mathbf{e}_i^T,$$

where the orthogonal $\mathbf{e}_i$ are $\lambda_i^{1/2}\mathbf{l}_i$, called "error patterns". An example of the error patters of retrieval noise covariance of ozone is shown in Figure 2.

Figure 2.: Error patters of the retrieval covariance of Ozone.



The nonlinearity parameter $c^2 = \delta\mathbf{y}^T\mathbf{S}_\varepsilon^{-1}\delta\mathbf{y}$ for the ten largest error patterns of the retrieval covariance is shown in the tables for pressure and temperature, water vapor and ozone (MIPAS retrieved profiles of 20020925, 03:10:57, 50.99 lat. -74.12 lon).

The comparison is evaluated in the MIPAS microwindows (selected spectral intervals).

Four cases are examined: effect due only to the retrieved species (worst case) and due to all species (real case), coming from the full retrieval range and from the MIPAS Occupation Matrices OM defined as [microwindows, altitudes] optimal selection.

The values of the nonlinearity parameter are all much less than unity, indicating that within the range of the solution error the problem is nearly linear.

Pressure and Temperature Non linearity

Error Pattern	Eigenv. $\lambda^{1/2}$	All hgt, only CO_2	All hgt, all gases	OM, only CO_2	OM, all gases
1	2.74	0.00094	0.00088	0.00151	0.00125
2	2.42	0.00170	0.00145	0.00383	0.00313
3	2.20	0.00692	0.00631	0.01439	0.01248
4	1.87	0.00106	0.00108	0.00182	0.00182
5	1.52	0.00049	0.00053	0.00087	0.00090
6	1.41	0.00066	0.00055	0.00129	0.00100
7	1.35	0.00089	0.00086	0.00123	0.00108
8	1.20	0.00766	0.00705	0.00701	0.00662
9	1.08	0.00142	0.00204	0.00219	0.00296
10	0.97	0.00073	0.00072	0.00139	0.00134

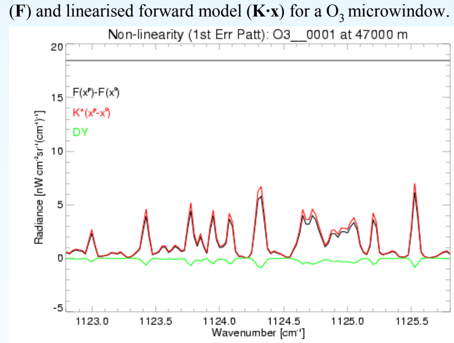
H_2O Non linearity

Error Pattern	Eigenv. $\lambda^{1/2}$	All hgt, only H_2O	All hgt, all gases	OM, only H_2O	OM, all gases
1	0.68	0.00029	0.00029	0.00035	0.00034
2	0.55	0.00012	0.00012	0.00016	0.00016
3	0.46	0.00011	0.00011	0.00009	0.00009
4	0.45	0.00015	0.00014	0.00048	0.00043
5	0.39	0.00043	0.00041	0.00050	0.00044
6	0.37	0.00010	0.00009	0.00030	0.00027
7	0.34	0.00012	0.00012	0.00009	0.00008
8	0.30	0.00002	0.00002	0.00003	0.00003
9	0.26	0.00002	0.00002	0.00002	0.00002
10	0.23	0.00003	0.00003	0.00004	0.00004

O_3 Non linearity

Error Pattern	Eigenv. $\lambda^{1/2}$	All hgt, only O_3	All hgt, all gases	OM, only O_3	OM, all gases
1	0.57	0.02522	0.02515	0.07716	0.07709
2	0.40	0.00139	0.00135	0.00046	0.00043
3	0.31	0.00052	0.00049	0.00032	0.00029
4	0.24	0.00103	0.00095	0.00315	0.00308
5	0.19	0.00134	0.00104	0.00206	0.00176
6	0.16	0.00334	0.00257	0.00341	0.00264
7	0.14	0.03022	0.02126	0.02964	0.02070
8	0.11	0.00207	0.00141	0.00180	0.00114
9	0.10	0.00397	0.00287	0.00396	0.00287
10	0.09	0.00009	0.00007	0.00008	0.00006

Figure 3: Worst example of comparison between forward model (**F**) and linearised forward model (**K·x**) for a O_3 microwindow.



CONCLUSIONS

- MIPAS forward model is nearly linear within the error bounds of real retrieved data
- Assimilation of linearised measurements or of the information content could be constructed

Next Step

Assimilation of MIPAS data at DARC using this alternative approach

- ➢Linearisation of the forward model
- ➢Averaging kernel representation of the retrieval

References

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REFERENCE FORWARD MODEL, http://www.atm.ox.ac.uk/RFM/

A. Dudhia, V. L.Jay and C.D.Rodgers, *Microwindows selection for High Spectral Resolution Sounders,* Appl. Optics, **41**, 3665 (2002).