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***An Atmospheric Radiative
Transfer Primer***

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Acknowledgements

Preface

The aim of this text is to provide an initial introduction to atmospheric radiative transfer. I have attempted to link the undergraduate electromagnetic wave description of light to the radiometric methods used in more advanced courses. I have ignored or cursorily dealt with many aspects of this very broad topic to try and give the reader a vade mecum of the most relevant formula and techniques.

The mks system of units [*BIPM*, 2006] is used in this book. I have occasionally appended to equations the dimensions in mks units to make the result of a formula more apparent.

The following mathematical conventions have been adopted. Scaler variables are given in italic lower-case type. Scaler constants are given in italic upper-case type. Vectors are given in bold type using lower case symbols except where they represent fields in which case they are capitalised. The magnitude of a vector uses the same symbol but in italic type. Matrices are given in bold upper-case type.

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Nomenclature

Constants

σ Stephan-Boltzmann constant
 c velocity of light
 h Planck's constant
 k_B Boltzmann's constant
 p_0 standard pressure
 A_N Avogadro's number
 T_0 standard temperature

Variables

α_C collision (or pressure) broadening half-width
 α_D Doppler broadening e^{-1} -width
 α'_D Doppler broadening half-width
 β^{abs} volume absorption coefficient
 β^{ext} volume extinction coefficient
 β^{sca} volume scattering coefficient
 ϵ emissivity
 θ zenith angle
 λ wavelength
 μ cosine of the zenith angle
 $\tilde{\nu}$ wavenumber
 ν frequency
 ν_0 line centre
 ϕ azimuth angle

ρ mass density (mass per unit volume)
 σ^{abs} absorption cross section
 σ^π backscatter cross section
 σ^{ext} extinction cross section
 τ optical depth
 $\tilde{\omega}$ single scatter albedo
 $d\omega$ elemental solid angle
 Φ radiant flux
 Φ_λ spectral radiant flux
 $d\Omega$ elemental solid angle projection
 k complex part of refractive index
 k^{abs} mass absorption coefficient
 k^{ext} mass extinction coefficient
 k^{sca} mass scattering coefficient
 m mass of a body or air parcel
 m_m mass of a single atom or molecule
 m complex refractive index
 n number density of atoms or molecules, particles per unit volume
 n real part of refractive index
 n_m mole
 r radius
 p pressure
 t time

Nomenclature

xiii

v velocity

A area

B_λ spectral radiance from a blackbody

E irradiance

F shape factor

F_C shape factor for collision broadening

F_D shape factor for Doppler broadening

I radiant intensity

\bar{I} average radiant intensity

L radiance

L^b radiance from a blackbody

M mass of a mole of a substance

M radiant exitance

M^b radiant exitance from a blackbody

N number of atoms or molecules

P phase function

Q radiant energy

Q^{scat} scattering efficiency factor

Q^{abs} absorption efficiency factor

Q^{ext} extinction efficiency factor

S line strength

T temperature

T_B brightness temperature

Some Comments on Notation

The notation adopted here attempts to make the various redirections of energy clear by ordering the function dependence on angular terms as input direction(s) followed by out going direction(s). Multiple terms are ordered so the redirection of radiation should be apparent by reading the combination of terms from left to right. The subscripts 'i', 'r' and 't' are used to denote incident, reflected and transmitted terms. In addition ω is used to refer to a specific direction while 2π is used to note a hemisphere. For example the incident irradiance from a specific direction is denoted $E(\omega_i)$ while irradiance from the hemisphere is denoted by $E(2\pi)$. It follows that in this case $E(2\pi) = \cos \theta_i E(\omega_i)$.

Terms describing the transfer of energy are spectral in nature however to save space this dependence has not been explicitly shown except where the term is not monochromatic.