

## Isentropic Dynamical Interpolation

The analysis is defined as the field which minimises the following quadratic penalty function:

$$\begin{aligned}\mathcal{J} = & w_{\text{ap}} \int \left[ \chi_t + \mathbf{u} \cdot \nabla \chi \right]^2 dA \\ & + w_{\text{num}} \int \left[ c_1 \frac{\partial^2 \chi}{\partial t^2} + \nabla^2 \chi \right]^2 dA \\ & + w_{\text{obs}} \sum \frac{\left[ \chi_i - \chi(\lambda_i, \phi_i, t_i) \right]^2}{\sigma_i^2},\end{aligned}$$

where  $\chi$  is the species mixing ratio,  $\mathbf{u}$  is the horizontal wind taken from ECMWF operational analyses,  $\chi_i$  and  $\sigma_i$  are the observations and noise estimates interpolated onto the isentropic surface.  $w_{\text{ap}}$ ,  $w_{\text{num}}$ ,  $c_1$  and  $w_{\text{obs}}$  are constants.