relative_azimuth-surface

July 17, 2020

This notebook examines the use of relative azimuth in our calculation of surface reflectance.

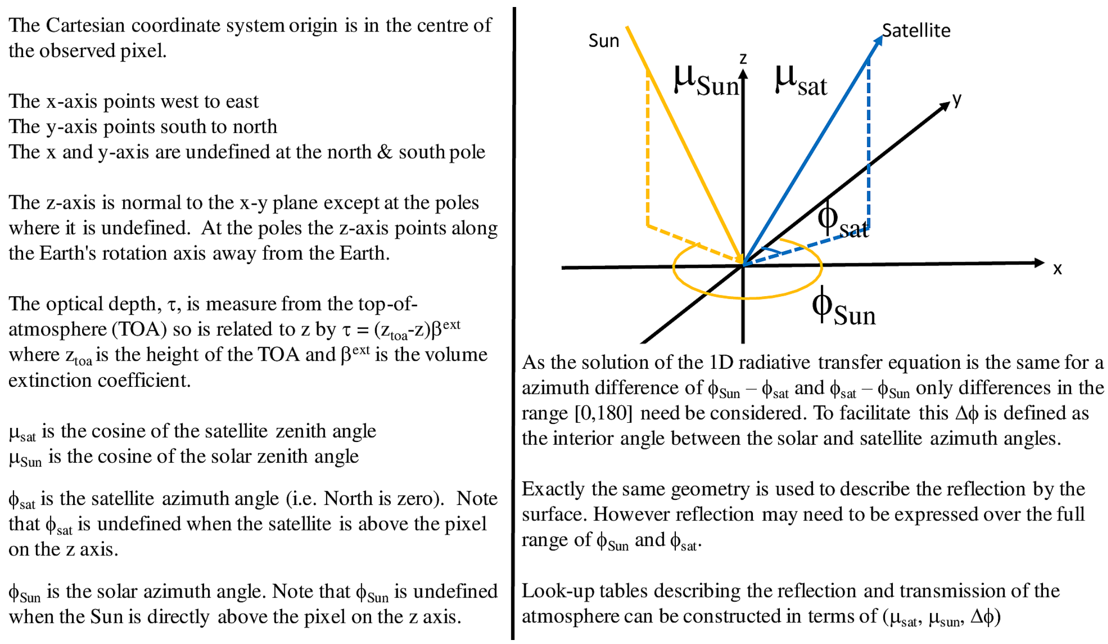

Figure 1: A definition of azimuth and zenith angles by Don.

Due to radiative transfer's legacy in astronomy, this coordinate system differs from the traditional spherical polar system in two ways: - Azimuth angles are measured clockwise (looking down the z-axis) from the y-axis rather than the more traditional anti-clockwise from the x-axis. - The zenith angle of the solar beam is 0° at noon rather than the more traditional 180°. (The dot product of the upwards z and downwards illumination would be -1.)

Over sea, relative azimuth is passed to the Cox-Munk scheme, which impliments our sea surface reflectance model, as outlined in Sayer et al. (2010) (hereafter S10). That used to invert the angle whenever it is used (though it's worth pointing out that the only time we use relazi and aren't integrating over quadrature points in $[0, 2\pi]$ is in the calculation of ρ_{0v}), but Simon has since reverted that. The model combines three components to calculate the direct beam reflectance,

$$R_{dd} = f_{wc}\rho_{wc} + (1 - f_{wc})(\rho_{gl} + \rho_{ul}).$$

Firstly, there is reflection off whitecaps and other wind-generated foam. This is calculated here as the product of the fractional cover of whitecaps and their reflectance. The optimal-least-squares parameterisation of Monahan and Muircheartaigh (1980) is used to estimate the fractional cover of whitecaps,

$$f_{wc} = 2.951 \times 10^{-6} w^{3.52},$$

for a 10 m wind speed of w ms^{-1}. (Note that we use ECMWF's 2 m wind speed.)

The reflectance of whitecaps ρ_{wc} has been specified as,

λ (nm)	470	550	650	870	1,240	1,375	1,600	2,130	3,700
	0.4408	0.4024	0.3544	0.2488	0.0712	0.0064	0	0	0
	0.44	0.40	0.35	0.25	0.0712	-	0.06	0	0
	0.40	0.40	0.40	0.24	0.0712	-	0.06	0	0

The first row gives the current values, as calculated by an unspecified "external program". The second row gives values that Greg added in 2015. The third row gives the values used previous to that, as specified in S10:

> Koepke (1984) treated whitecap reflectance in the visible region as constant with wavelength, although noted that in the near-infrared it might be expected to decrease due to absorption by water molecules. More recent coastal (Frouin et al., 1996) and open ocean (Nicolas et al., 2001) work suggests a reflectance of about 0.4 at shorter wavelengths, decreasing by about 40% at 850 nm and 85% at 1.65 μm. These ratios have been adopted here for use at the nearby (A)ATSR channels, with reflectance at 550 nm and 660 nm assumed equal to 0.4.

I have no scientific opinion about the reflectance at 1.6μm but am intrigued that **we quietly switched from 0.06 to zero**.

Secondly, there is the underlight: light scattered from the ocean depths. For deep Case I waters, we assume the transmistance of the abyss is negligible and approximate this reflectance by a infinite series of reflections in the uppermost layer of the sea,

$$\rho_{ul} = \frac{T_d R_w T_u}{1 - R_u R_w}.$$

Here,

- The downwelling transmittance coefficient $T_d = 1 - R_f(\theta_s)$, where R_f is the Fresnel reflectivity coefficient which will be defined later.
- The upwelling transmittance coefficient $T_u = 0.52$ from a numerical integration of S10's Eq. 21. (Spectral variation of order 0.01 is neglected.)
- The upwelling reflectance coefficient $R_u = 1 - T_u$.
- The water body reflectance $R_w = fb(\lambda)/a(\lambda)$, where a and b are the extinction and backscatter coefficients of sea water. A subscript of w denotes pure water. We either draw those values from monthly averages of the Ocean Colour CCI product or assume the following values,

λ (nm)	470	550	650	870	1,240	1,375	1,600	2,130	3,700
a	0.02694	0.06585	0.3518	5.365	359.9	1,115	671.5	3,380	12,230
a_w	0.016	0.064	0.35	5.365*	359.9	1,115	671.5	3,380	12,230
$b \times 10^{-4}$	37.61	25.94	18.79	12.39	8.667	7.944	7.056	5.771	4.188
$b_w \times 10^{-7}$	3,780	1,930	9379	2662	575.9	368.5	191.5	55.63	5.12

*This value is given as 5.65 in S10's Tab. 4.

- The constant of proportionality was given by Morel and Gentili (1991) as,

$$f = 0.6279 - 0.2227\eta_b - 0.00513\eta_b^2 + (0.2465\eta_b - 0.3119)\cos\theta_s,$$

where $\eta_b = b_w/b$, the ratio of backscattering that is due to the water alone.

Thirdly, there is glint reflectance, which follows Cox and Munk (1954) (hereafter C&M). That paper introduces its geometry with a "useful" figure. I follow that by a superior figure from Zhang and Wang (2010) (herafter Z&W), a review of sea surface reflectance parameterisations, which uses slightly different notation. The proofs to follow will relate to the formulations of all three papers, with C&M variables in black, Z&W variables in red, and S10 variables in blue.

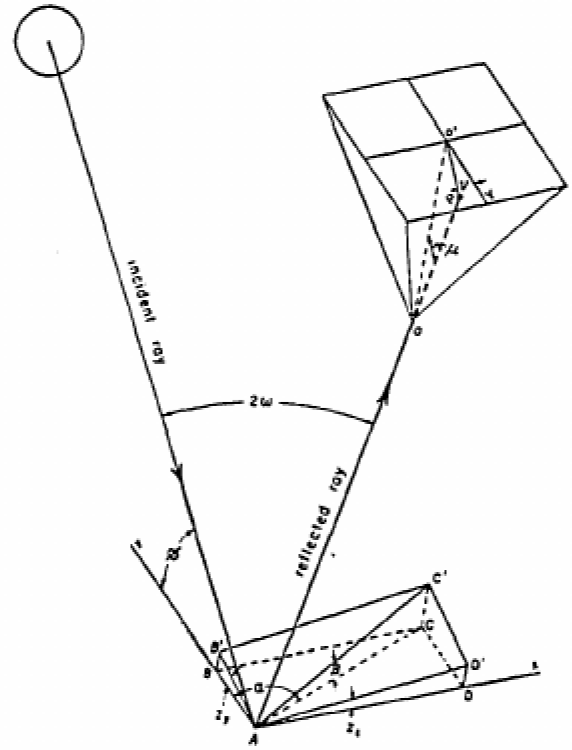

FIG. 2. The coordinate system is centered at the sea surface with the z-axis vertically upward (not shown) and the y-axis drawn horizontally toward the sun. The incident ray is reflected at A and forms an image at P on a horizontal photographic plate. Points A, B, C, and D define a horizontal plane through A and $AB'C'D'$, the plane tangent to the sea surface. The tilt β is measured in the direction AC of steepest ascent, and this direction makes an angle α to the right of the sun. OO' is parallel to the z-axis and $O'Y'$ to the (negative) y-axis.

Figure 2: An excerpt from Cox and Munk's first paper of 1954 on surface inclination which introduces their angular variables.

Fig. 2 defines a y-axis pointing at the sun and vertical z-axis. (x points right to complete a right-handed coordinate system.) The Sun has *elevation* angle ϕ. The reflected beam is observed with *zenith* angle μ and azimuth ν clockwise (looking down the z-axis) from the negative y-axis. The angle between the incident and reflected rays is 2ω. The direction of steepest ascent of the surface has azimuth α clockwise from the solar direction and *elevation* β.

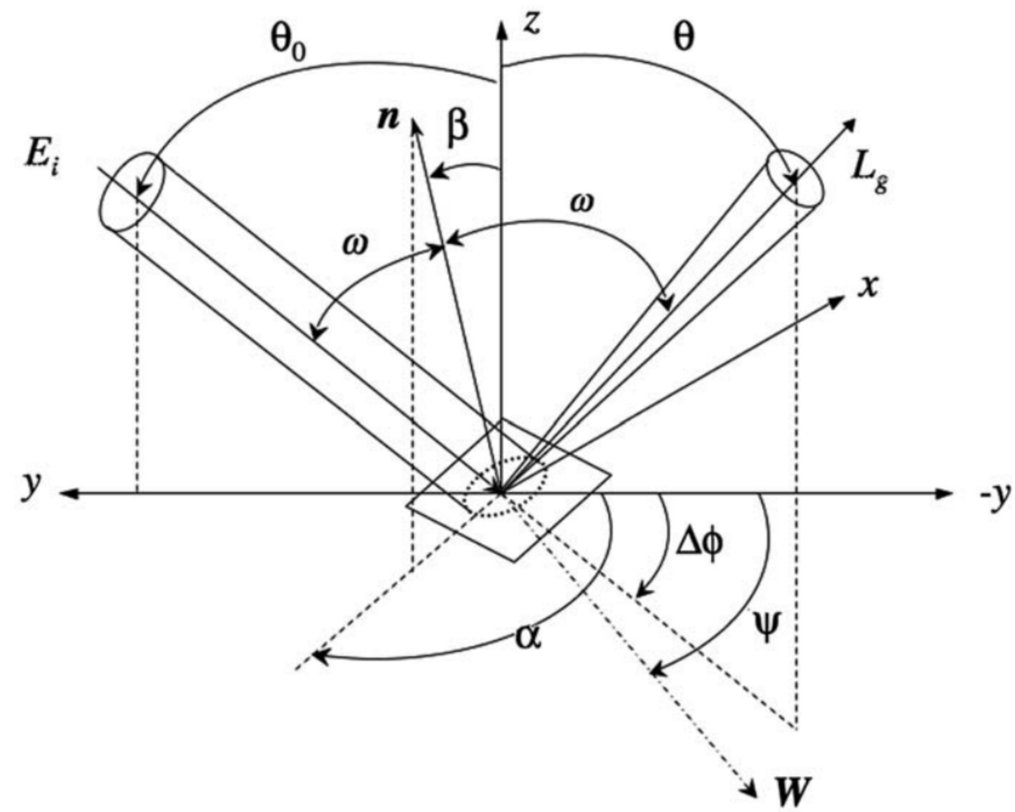

Fig. 1. Scattering configurations used in sun glitter models.

Figure 3: An except from Zhang et al. (2010) that presents a similar coordinate system to Cox and Munk but with much more legible labelling.

Fig. 3 uses a solar *zenith* θ_0, a view *zenith* θ, and a view azimuth $\Delta\phi$ (again clockwise looking down the z-axis from the negative y-axis). The normal to the surface has a zenith angle β and azimuth α (using the same reference as the view azimuth), which are equivalent to the angles C&M define from the line of steepest ascent.

Consider an infinitesimal horizontal area Δ_h of the sea surface. Using C&M's notation,

- The actual area of that surface will be $\Delta_h \sec\beta$.
- The area normal to the Sun will be $\Delta_h \sec\beta \cos\omega$.
- If this area receives irradiance δH from a point on the Sun and has reflection coefficient ρ, the reflected flux is $\rho\, \delta H \Delta_h \sec\beta \cos\omega$.
- By the reciprocity of reflection, this is radiated into a solid angle $\pi\epsilon^2$ equal to that subtended by the sun at the surface with intensity

$$\delta J = \rho \delta H \Delta_h (\pi\epsilon)^{-1} \sec\beta \cos\omega.$$

- As the solid angle subtended by the observer is much smaller than that subtended by the Sun, we can simply sum over the surface of the Sun and $\delta H \to H$. Thus, the reflected intensity from this area of the sea is (C&M's Eq. 7),

$$J = \frac{\rho \Delta_h H}{\pi \epsilon^2} \sec\beta \cos\omega.$$

The law of reflection requires that only a small range of surface inclinations will reflect light towards the observer. If there are n appropriately aligned surface facets in an area A, the radiant flux reflected towards the observer will be (C&M's Eq. 8),

$$N = \frac{n}{A} J \sec\mu = \frac{P}{\Delta_h} J \sec\mu,$$

where P is the fraction of the area that is appropriately aligned. C&M §4.1 argues that $P = \frac{\pi\epsilon^2}{4}\sec^3\beta \sec\omega\, p(Z_x, Z_y)$, where p is the probability of observering a sea surface with horizontal and vertical gradients Z_x, Z_y. Thus, the reflected solar intensity is,

$$N = \frac{H}{4}\rho \sec^4\beta\, p(Z_x, Z_y) \sec\mu.$$

The reflectance of the sea surface is then π times the ratio of N to the nadir solar intensity $H\sin\phi$,

$$\rho_{gl} \equiv \frac{\pi N}{H \sin\phi} = \frac{\pi}{4}\rho \sec^4\beta\, p(Z_x, Z_y) \sec\mu \csc\phi.$$

We can convert this to the notation used in Z&W through the transforms $\phi \to \frac{\pi}{2} - \theta_0, \mu \to \theta, \nu \to \Delta\phi, \alpha \to \alpha, \beta \to \beta, \omega \to \omega, \rho \to R(\omega)$:

$$\rho_{gl} = \frac{\pi R(\omega) p(Z_x, Z_y)}{4\cos^4\beta \cos\theta \cos\theta_0}.$$

which replicates Z&W's Eq. 4.

Alternatively, we can convert this expression to the standard ORAC notation through $\phi \to \frac{\pi}{2} - \theta_s, \mu \to \theta_v, \nu \to \phi_r - \pi, \beta \to \beta, \omega \to \Theta, \rho \to R_f$:

$$\rho_{gl} = \frac{\pi R_f(\Theta) p(Z_x, Z_y)}{4\cos^4\beta \cos\theta_v \cos\theta_s},$$

which replicates S10's Eq. 12.

One last alteration is made to this expression. Following Zeisse (1995), wherever the satellite zenith angle exceeds 70° with a wind speed $> 1\,\mathrm{ms}^{-1}$ the term $\cos\theta_v$ in the denominator is replaced by,

$$\cos\theta_v + (1, t, t^2)^T \mathbf{C} \begin{pmatrix} 1 \\ w_p \\ w_p^2 \end{pmatrix},$$

where $t = (\theta - 70°)/5$, $w_p = 4\log w/1.30103$ for a wind speed $w\,\mathrm{ms}^{-1}$, and $\mathbf{C}$ is a set of nine empirical constants.

The Fresnel reflectivity of unpolarized light is,

$$R_f(\omega) = \frac{1}{2}\left[\frac{\sin^2(\omega - \omega')}{\sin^2(\omega + \omega')} + \frac{\tan^2(\omega - \omega')}{\tan^2(\omega + \omega')}\right],$$

where $n_{wat}\sin\omega' = n_{air}\sin\omega$ by Snell's Law for refractive indices n. We assume $n_{air} = 1.00029$ and,

λ (nm)	470	550	650	870	1,240	1,375	1,600	2,130	3,700
n_{wat} (-)	1.345	1.341*	1.338*	1.334*	1.327	1.325	1.323*	1.313	1.374

These appear to have remained constant for the last decade. The stared values are drawn from Tab. 2 in S10. The paper states,

> These were calculated at 550 nm and 670 nm using the method of Quan and Fry (1995) assuming a temperature of 15°C and salinity of 35 parts per thousand, but are correct to four significant figures over the range of typical temperatures and salinities. This model of Quan and Fry (1995) extends only to 700 nm, so for the longer wavelengths values for pure water from Hale and Querry (1973) were used. At shorter wavelengths there was an offset of around 0.0065 between the refractive index as predicted for pure water and that of salinity typical for the sea, so this adjustment was also applied to the pure water data used at 870 nm and 1.6 μm.

The first calculation is replicated below. The second paper has a rather more involved functional form, so I interpolate their Tab. 1.

[1]:
```
import numpy as np

l = np.array([470., 550., 650., 870., 1240., 1375., 1600., 2130., 3700.])
s = 35.
t = 15.

n0 = 1.31405
n1 = 1.779e-4
n2 = -1.05e-6
n3 = 1.6e-8
n4 = -2.02e-6
n5 = 15.868
n6 = 0.01155
n7 = -0.00423
n8 = -4382
n9 = 1.1455e6

print("Quan and Fry (1995)")
n = n0 + (n1 + n2*t + n3*t*t)*s + n4*t*t + (n5 + n6*s + n7*t)/l + n8/(l*l) + n9/
↪(l*l*l)
for lamb, n_r in zip(l, n):
    print(f"  n_r({lamb:4.0f} nm) = {n_r:6.4f}")
```

```
Quan and Fry (1995)
  n_r( 470 nm) = 1.3451
  n_r( 550 nm) = 1.3413
  n_r( 650 nm) = 1.3381
  n_r( 870 nm) = 1.3340
  n_r(1240 nm) = 1.3302
```

```
  n_r(1375 nm) = 1.3293
  n_r(1600 nm) = 1.3281
  n_r(2130 nm) = 1.3262
  n_r(3700 nm) = 1.3235
```

[2]:
```
l0 = np.array([
    0.200, 0.225, 0.250, 0.275, 0.300, 0.325, 0.350, 0.375, 0.400, 0.425, 0.450,
    0.475, 0.500, 0.525, 0.550, 0.575, 0.600, 0.625, 0.650, 0.675, 0.700, 0.725,
    0.750, 0.775, 0.800, 0.825, 0.850, 0.875, 0.900, 0.925, 0.950, 0.975, 1.0, 1.
 ↪2,
    1.4, 1.6, 1.8, 2.0, 2.2, 2.4, 2.6, 2.65, 2.70, 2.75, 2.80, 2.85, 2.90, 2.95,
    3.00, 3.05, 3.10, 3.15, 3.20, 3.25, 3.30, 3.35, 3.40, 3.45, 3.50, 3.6, 3.7,␣
 ↪3.8
])
n0 = np.array([
    1.396, 1.373, 1.362, 1.354, 1.349, 1.346, 1.343, 1.341, 1.339, 1.338, 1.337,
    1.336, 1.335, 1.334, 1.333, 1.333, 1.332, 1.332, 1.331, 1.331, 1.331, 1.330,
    1.330, 1.330, 1.329, 1.329, 1.329, 1.328, 1.328, 1.328, 1.327, 1.327, 1.327,
    1.324, 1.321, 1.317, 1.312, 1.306, 1.296, 1.279, 1.242, 1.219, 1.188, 1.157,
    1.142, 1.149, 1.201, 1.292, 1.371, 1.426, 1.467, 1.483, 1.478, 1.467, 1.450,
    1.432, 1.420, 1.410, 1.400, 1.385, 1.374, 1.364
])

print("Adjusted Hale and and Querry (1973)")
n_interp = np.interp(l*1e-3, l0, n0) + 0.0065
for lamb, n_r in zip(l, n_interp):
    print(f"  n_r({lamb:4.0f} nm) = {n_r:6.4f}")
```

```
Adjusted Hale and and Querry (1973)
  n_r( 470 nm) = 1.3427
  n_r( 550 nm) = 1.3395
  n_r( 650 nm) = 1.3375
  n_r( 870 nm) = 1.3347
  n_r(1240 nm) = 1.3299
  n_r(1375 nm) = 1.3279
  n_r(1600 nm) = 1.3235
  n_r(2130 nm) = 1.3060
  n_r(3700 nm) = 1.3805
```

Hence, I'm not entirely sure where the values above 870nm in the code come from.

A central result of C&M is their empirical expression for the probability of observing a given sea-surface slope,

$$p(Z_x, Z_y) = \frac{1}{2\pi\,\sigma_x\sigma_y} \exp\left[-\frac{1}{2}\left(\frac{Z_x^2}{\sigma_x^2} + \frac{Z_y^2}{\sigma_y^2}\right)\right],$$

where $\sigma_x^2 = 0.003 + 0.00192w$ and $\sigma_y^2 = 0.00316w$. In this case, w is the wind speed (in ms^{-1})

41 feet above sea level. C&M multiplied this by a Gram-Charlier series, but Z&W found that the model is more accurate without it.

C&M then defines three normalised vectors: the normal to the surface, the incident ray, and the reflected ray, respectively,

$$\hat{\mathbf{n}} = \begin{pmatrix} -\sin\alpha\sin\beta \\ -\cos\alpha\sin\beta \\ \cos\beta \end{pmatrix}, \qquad \hat{\mathbf{i}} = \begin{pmatrix} 0 \\ -\cos\phi \\ -\sin\phi \end{pmatrix}, \qquad \hat{\mathbf{r}} = \begin{pmatrix} -\sin\nu\sin\mu \\ -\cos\nu\sin\mu \\ \cos\mu \end{pmatrix}. \tag{1}$$

The law of reflection requires that the difference between the incident and reflected beams is parallel to the surface normal such that,

$$\hat{\mathbf{r}} - \hat{\mathbf{i}} = \begin{pmatrix} -\sin\nu\sin\mu \\ -\cos\nu\sin\mu + \cos\phi \\ \cos\mu + \sin\phi \end{pmatrix} = C \begin{pmatrix} -\sin\alpha\sin\beta \\ -\cos\alpha\sin\beta \\ \cos\beta \end{pmatrix}, \tag{2}$$

We find the arbitrary constant C by considering the magnitude of these vectors,

$$C^2 = \sin^2\nu\sin^2\mu + \cos^2\nu\sin^2\mu + \cos^2\phi - 2\cos\nu\sin\mu\cos\phi + \cos^2\mu + \sin^2\phi + 2\cos\mu\sin\phi \tag{3}$$
$$= 2(1 + \cos\mu\sin\phi - \cos\nu\sin\mu\cos\phi). \tag{4}$$

The angle ω between the incident and reflected rays is defined in Figs. 1 and 2. Considering the negative of the incident vector as both vectors leave the same point,

$$\hat{\mathbf{r}} \cdot (-\hat{\mathbf{i}}) = 0 - \cos\nu\sin\mu\cos\phi + \cos\mu\sin\phi = \cos 2\omega, \tag{5}$$

using the identity $\mathbf{a} \cdot \mathbf{b} = |a||b|\cos\Omega$. Thus,

$$C^2 = 2(1 + \cos 2\omega) \tag{6}$$
$$C = \pm 2\cos\omega. \tag{7}$$

C&M's Eq. 4 then comes from the components of (2). Specifically, the z-components gives,

$$2\cos\omega\cos\beta = \cos\mu + \sin\phi, \tag{8}$$

or Eq. 2 of Z&W,

$$\cos\beta = \frac{\cos\theta + \cos\theta_0}{2\cos\omega}, \tag{9}$$

or Eq. 13 of S10,

$$\cos\beta = \frac{\cos\theta_v + \cos\theta_s}{2\cos\Theta} = \frac{\cos\theta_v + \cos\theta_s}{\sqrt{2 + 2\cos 2\Theta}}, \tag{10}$$

The ratio of the y and x-components of (2) gives,

$$\frac{-\cos\nu\sin\mu}{-\sin\nu\sin\mu} = \frac{-2\cos\omega\cos\alpha\sin\beta - \cos\phi}{-2\cos\omega\sin\alpha\sin\beta} \tag{11}$$
$$\cot\nu = \cot\alpha + \frac{1}{2}\cos\phi\sec\omega\csc\alpha\csc\beta, \tag{12}$$

which **differs from the published C&M Eq. 4 in the sign of the last term**. The corresponding expression in Z&W is Eq. 3, which derives from a rearrangement of the y-component of (2),

$$\cos\alpha = \frac{\cos\nu\sin\mu - \cos\phi}{2\cos\omega\sin\beta} \tag{13}$$

$$\cos\alpha = \frac{\cos\Delta\phi\sin\theta - \sin\theta_0}{2\cos\omega\sin\beta}. \tag{14}$$

I'm not worrying about the sign error because I'm not aware of anyone using C&M's direct forumaltion in code and I replicate every previous line of their derivation. S10 does not give an explicit expression for α.

Expressions for the scattering angle are derived by transforming (5) to give Z&W's Eq. 1,

$$\cos 2\omega = \cos\theta\cos\theta_0 - \cos\Delta\phi\sin\theta\sin\theta_0. \tag{15}$$

or S10's Eq. 14,

$$\cos 2\Theta = \cos\theta_s\cos\theta_v + \cos\phi_r\sin\theta_s\sin\theta_v. \tag{16}$$

Thus, a straightforward way to identify the angle standard being used is to examine the sign in this expression for the scattering angle. We desire it to be positive (being what we do).

The slope of the surface is found by considering the segment AD in Fig. 2 to be an infinitesimal length δx, AB to be δy, and CC′ to be δz. Then,

$$\sin\alpha = \delta x/D \qquad \cos\alpha = \delta y/D \qquad \tan\beta = \delta z/D, \tag{17}$$

where $D = \sqrt{\delta x^2 + \delta y^2}$ is the length of the horizontal diagonal AC. Multiplying the first and last of these for constant y,

$$\sin\alpha\tan\beta = \lim_{\delta x,y,z\to 0}\frac{\delta x\delta z}{\delta x^2 + \delta y^2} \tag{18}$$

$$= \left.\frac{\partial z}{\partial x}\right|_y = Z_x. \tag{19}$$

Similarly,

$$Z_y = \left.\frac{\partial z}{\partial y}\right|_x = \cos\alpha\tan\beta, \tag{20}$$

reproducing C&M's Eq. 1.

We find Z in terms of the viewing geometry by taking the ratio of the x and z-components of (5) while making use of (18),

$$\frac{-\sin\nu\sin\mu}{\cos\mu + \sin\phi} = \frac{-\sin\alpha\sin\beta}{\cos\beta} \tag{21}$$

$$= -Z_x. \tag{22}$$

Similarly from the ratio of the y and z-components,

$$Z_y = \frac{\cos\nu\sin\mu - \cos\phi}{\cos\mu + \sin\phi}. \tag{23}$$

In Z&W's approach, these expressions can be recovered by combining their Eq. 1-3,

$$\begin{aligned} Z_x &= \sqrt{1-\cos^2\alpha}\tan\beta & (24)\\ &= \frac{\sin\Delta\phi\sin\theta}{\cos\theta+\cos\theta_0} & (25)\\ Z_y &= \cos\alpha\tan\beta & (26)\\ &= \frac{\cos\Delta\phi\sin\theta-\sin\theta_0}{2\cos\omega\sin\beta}\tan\beta & (27)\\ &= \frac{\cos\Delta\phi\sin\theta-\sin\theta_0}{2\cos\omega}\frac{2\cos\omega}{\cos\theta+\cos\theta_0} & (28)\\ &= \frac{\cos\Delta\phi\sin\theta-\sin\theta_0}{\cos\theta+\cos\theta_0}, & (29) \end{aligned}$$

where a depressing amount of algebra goes into the Z_x derivation.

We can also convert (21) into S10 notation,

$$\begin{aligned} Z_x &= -\frac{\sin\phi_r\sin\theta_v}{\cos\theta_v+\cos\theta_s} & (30)\\ Z_y &= -\frac{\cos\phi_r\sin\theta_v+\sin\theta_s}{\cos\theta_v+\cos\theta_s}, & (31) \end{aligned}$$

which replicates their Eq. 7-8 **except for the sign on the y derivative**. According to Andy Sayer, this expression was taken from the manual for the 6s RT code (page 12, the explanation of OCEABRDF) and I can confirm that the positive expression for Z_y is implimented in that code.

To understand that difference, consider when $\theta_s = \theta_v = 45°$,

$$\begin{pmatrix} Z_x \\ Z_y \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}\sin\phi_r \\ \frac{1}{2}(1+\cos\phi_r) \end{pmatrix}. \quad (32)$$

Thus, when the relative azimuth is zero, $Z = (0,1)^T$. That is a one-in-one slope, which is lowest towards the south. That would reflect sunlight *coming from the south* back along it's original trajectory. When the relative azimuth is 180°, $Z = (0,0)^T$, which is a flat surface for a simple reflection. Hence, these expressions use the desired relative azimuth convention.

When the relative azimuth is 90°, $Z = \frac{1}{2}(-1,1)^T$. That is a one-in-two slope which is lowest in the south-east. Again, if the sunlight is *coming from the south*, this is an appropriate surface to reflect it along the positive x-axis, which would be a relative azimuth of 90° clockwise from the y-axis. Further, in that manual the y-axis is defined as "pointed at the sun direction", which isn't inconsistent with a y-axis pointing in the same direction as sunlight. However, S10 transcribed the line as "in the direction of the sun", which is the source of our confusion. An inverted y-axis is also consistent with the closest the 6S manual comes to sketching it's coordinate system, at the top of p.12 of Part 1:

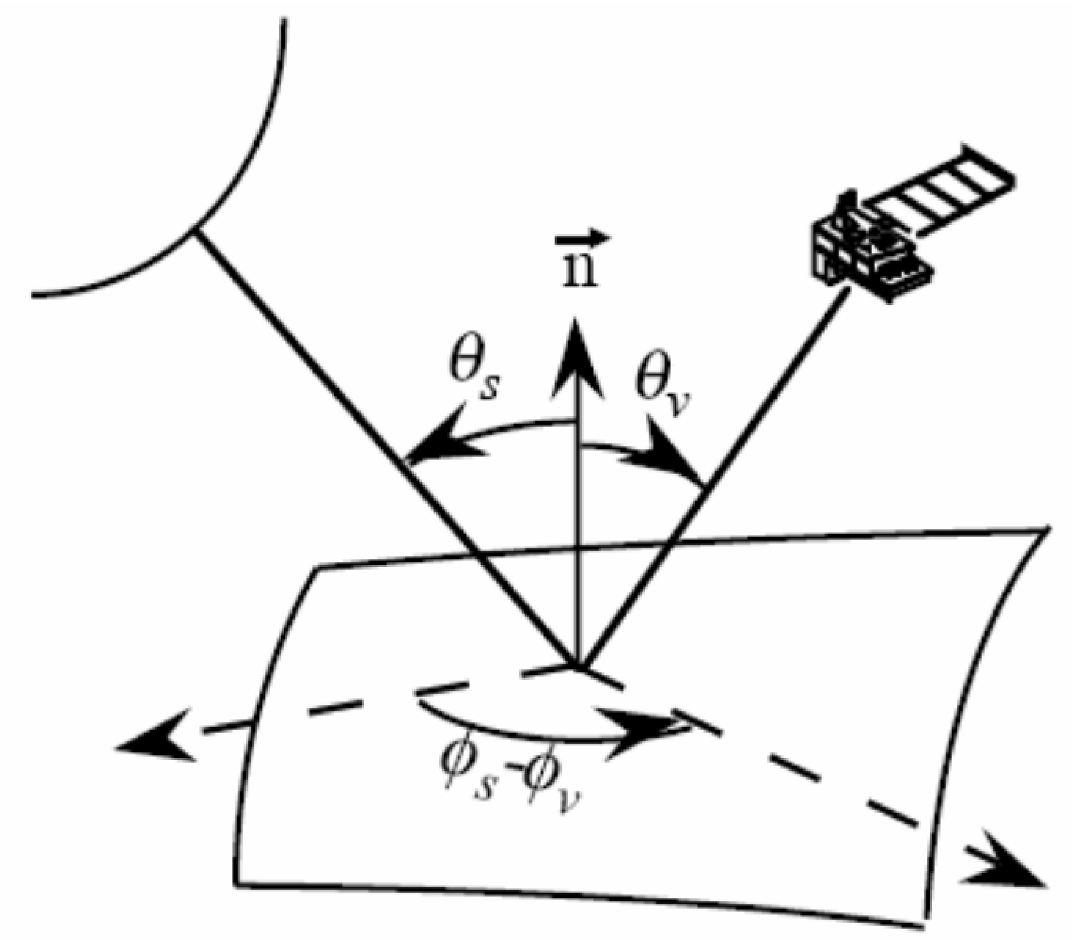

Figure 3: A sketch from the 6S user manual that is assumed to represent their coordinate system which points the y-axis away from the Sun.

Finally, C&M then tell the reader to rotate the axes to align the y-axis with the direction of the wind ("pointing upwind"), though this isn't explicitly shown. Z&W has equivalent expressions in its Eq. 6-8, but replaces α with $\alpha - \psi$, for a wind azimuth ψ (also defined from the negative y-axis).

$$Z'_x = \sin(\alpha - \psi) \tan \psi = \cos \psi Z_x - \sin \psi Z_y \tag{33}$$
$$Z'_y = \cos(\alpha - \psi) \tan \psi = \sin \psi Z_x + \cos \psi Z_y. \tag{34}$$

For a wind azimuth of 90°, the primed coordinates will therefore align with $-Z_y$ and Z_x, respectively. Looking at Fig. 3, that indeed corresponds to a right-handed coordinate system with the y'-axis pointing in the opposite direction to the wind aximuth.

In Eq. 9-10, S10 continues to follow the 6S manual by using the standard expression for a rotation of coordinate system by the wind's relative azimuth to the sun $\chi_w = \phi_s - \phi_w$,

$$Z'_x = \cos \chi_w Z_x + \sin \chi_w Z_y \tag{35}$$
$$Z'_y = - \sin \chi_w Z_x + \cos \chi_w Z_y. \tag{36}$$

In the unexpected coordinate system (e.g. in Fig. 3 the y-axis instead points right and the x-axis points out of the page), a wind blowing to the west (i.e. the hypothetical considered in the previous paragraph) would have $\phi_w = -90°$. When the sun is due north, $\chi_w = 90°$ and the primed coordinates align with Z_y and $-Z_x$, respectively. However, as the axes are defined in the opposite direction to Z&W, this is the desired result.

If we wish to use our standard coordinate system, one uses the negative Z_y expression from (31) and multiplies all subscript-y terms in (35-36) by -1. By reversing the definition of χ_w, we can return to the existing expression. This will practically change the sign of Z'_y but, as we neglect the Gram-Chalmer series, that term is only ever used after squaring so there will be no difference in the values output.

In conclusion,

- Our Cox-Munk implementation uses the desired relative azimuth convention of zero implying the sun and satellite are in the same part of the sky. Simon was correct to remove the converstions.
- The other expression for angles are correct.
- However, they use a different coordinate system to proofs in other parts of the code. Thus, for clarity, I would recommend that we put a minus sign in front Z_y and swap the definition of the relative wind azimuth.

The next use of relative azimuth is in `get_surface_reflectance()`.

For land surfaces, relative azimuth is passed to the calculation of the Ross-Thick Li-Sparse-Reciprocal BRDF kernels. This impliments the algorithm of Lucht et al. (2000) which references the kernels derived in Wanner and Stahler (1995), which itself references Roujean, Leroy, and Deschamps (1992). (That references a book by Ross from 1981 that is behind a paywall, so we won't worry about it.) The relative azimuth turns up in the phase function defined in the latter's Eq. (6),

$$P(\theta_s, \theta_v, \phi) = \frac{8}{3\pi} \frac{[(\pi - \xi)\cos\xi + \sin\xi]r + (-\xi\cos\xi + \sin\xi)t}{r + t} \tag{37}$$

$$\cos\xi = \cos\theta_s \cos\theta_v + \sin\theta_s \sin\theta_v \cos\phi, \tag{38}$$

where θ_s is the solar zenith, θ_v is the view zenith, ϕ is the relative azimuth, ξ is the phase angle of scattering, r is the reflectance, and t is the transmittance.

When $\phi = 0$, we see that $\cos\xi \equiv \cos(\theta_s - \theta_v)$ as our scattering angle ξ represents the difference between two zenith angles in the same plane. When $\phi = 180°$, we have $\cos\xi \equiv \cos(\theta_s + \theta_v)$ as the scattering angle is the sum of the zenith angles since they are on opposite sides of the normal. Further, when $\theta_s = \theta_v$ and $\phi = 0$ we see that $\cos\xi = 1$ and $P = \frac{8r}{3(r+t)}$, which is appropriate for specular reflection. Also, their Fig. 5 shows the Sun on the $\phi = 0$ axis.

Figure 4: Reproduction of Fig. 5 from Roujean, Leroy, and Deschamps (1992).

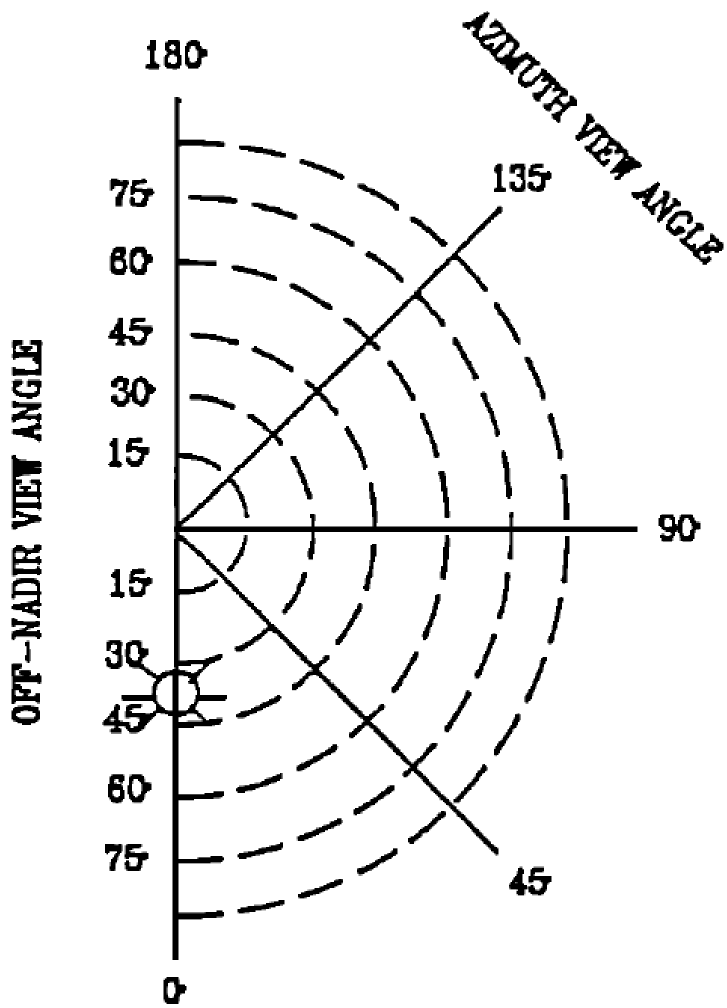

Fig. 5. Polar plot showing scheme for plotting bidirectional reflectance factors. The sun, indicated by sun symbol, is always located on the $\phi = 0°$ axis, the distance from the origin representing the sun zenith angle θ_s. The spectral bidirectional reflectance is defined in the polar plot, where the distance from the origin represents the off-nadir view angle θ_v, and the angle from $\phi = 0°$ represents the sensor's azimuth relative to the sun, ϕ. Curves in the polar plot appearing in Figures 6, 7, and 8 are isoreflectance curves.

These indicate that these equations use the same convention for relative azimuth as we desire. It is therefore appropriate that no alteration of the relative azimuth is made within `get_surface_reflectance()` land processing and `ross_thick_li_sparse.F90`.

Wthin DISORT, the azimuthal dependence of the radiative transfer equations is factored out by expression of the phase function as a sum of spherical harmonics, such that the only practical use of ϕ is in the construction of intensity from,

$$I(\tau, \mu, \phi) = \sum_{m=0}^{2M-1} I^m(\tau, \mu) \cos m(\phi_0 - \phi).$$

Thus, we would largely expect the convention for relative azimuth to be determined by how the phase function is decomposed. Technically, Fig. 3.3 of Thomas and Stamnes (1999) implies that they are using the standard spherical coordinate system, where azimuth is measured counter-clockwise from the x-axis but, as we only care about relative azimuths, this shouldn't matter. The only other use of the angle is in Eq. 2c of the DISORT manual which states that,

$$\cos \Theta = \mu\mu' + \sqrt{(1 - \mu^2)(1 - \mu'^2)} \cos(\phi - \phi'),$$

where the scattered beam is denoted by primes, Θ is the scattering angle, μ is a cosine of the zenith angle, and ϕ is an azimuth angle. Sadly, that is not what is implemented on line 2208 of DISORT.f

```
            CTHETA = -UMU0*UMU( IU ) + SQRT( ( 1.-UMU0**2 )*
     &                 ( 1.-UMU( IU )**2 ) )*COS( PHIRAD( JP ) )
```

In fact, they multiply μ_0 by -1 most times it is used throughout the code. This is consistent with their statement on page 52 of the manual, > polar (zenith) angles are measured from the upward direction: straight up is 0Âř and straight down is 180Âř (for historical reasons, the cosine of the incident beam angle is taken positive, whereas according to the DISORT convention all other downward-directed intensities have negative polar angle cosines)

Hence, **DISORT uses the opposite azimuth angle convention to what we desire**. (We shall leave the question of which convention *should* be used for another day.) Switching to our convention would make both terms on the right of the code above negative, meaning the scattering angle used here is $\pi - \omega$. In the surface proofs above, a scattering angle of zero meant the light returned along the incident path. In scattering theory, a scatter angle of zero means an unaltered direction of travel. Hence, the difference of definition.

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