Radiative Transfer Nomenclature, Symbols and Units

R.G. Grainger

June 11, 2009

List of Recommended Symbols

Symbol	Quantity	Latex Expression
ϕ	azimuth angle	
θ	zenith angle	
ω	solid angle	
Ω	projected solid angle	
ν	frequency	
λ	wavelength	
$\tilde{\nu}$	wavenumber	
ω	angular frequency	
k	angular wavenumber	
Q	energy	
Φ	radiant flux	
E	irradiance	
I	radiant intensity	
L	radiance	
$\mathcal{T}$	transmittance	
$\overline{\mathcal{T}}$	band transmittance	`\overline{{\cal T}}`
χ	optical path	`\chi`
τ	optical depth	`\tau`

Preface

The introduction "in the abstract" of so many new units, some rarely used and others misused, is not very palatable pedagogically.

Pedrotti et al. (2006)

Because atmospheric science cuts across a number of traditional disciplines in physics and chemistry, symbols nomenclature and units can be even more of a problem than usual.

Houghton (1979)

There is no standard system of symbols, terms, and units for radiative transfer quantities; each radiation subcommunity has its own distinctive and mutually incompatible traditions.

Petty (2004)

A good atmospheric radiative transfer text often has a paragraph in the *Preface* decrying the notation used by other authors. So this work is my attempt to define a standard set of names, symbols and units. Fortunately the wide adoption of the mkgs system means that units are not much of an issue, and there are only a few atmospheric terms where the nomenclature is unclear. It is the wide variation in the use of symbols that baffles the student and trips the expert. This document is a brief review of the symbols recommended by bodies such as the International Organization for Standardization as well as those that are commonly found in academic literature. Using these sources I have taken the step of recommending a standard set of symbols using guiding principles:

- generally,
  - scaler constants are represented by italic capitals,
  - scaler variables are represented by italic lower-case,
  - vectors are bold lower-case,
  - matrices are bold upper-case,
  - variables and constants in subscripts and superscripts should be italic, while textual labels should be in normal text fount;
- each symbol should represent a unique quantity,
- the adopted symbol should reflect common usage,
- given a near equal choice between symbols adopt the symbol recommended by the International Union of Pure and Applied Physics.

Contents

1 Standardising symbols

1.1 Existing Systems of Symbols and Units

1.1.1 Le Systeme International d'Unites

Perhaps the best known authority on units is Le Systeme International d'Unites (SI) which was established in 1960 at the Conference Generale des Poids et Mesures (CGPM). The base units of the International System are listed in Table 1, which relates the base quantity to the unit name and unit symbol for each of the seven base units *BIPM* (2006).

Base Quantity		SI Base Unit	
Name	**Symbol**	**Name**	**Symbol**
length	l, x, r, etc.	metre	m
mass	m	kilogram	kg
time, duration	t	second	s
electric current	I, i	ampere	A
thermodynamic temperature	T	kelvin	K
amount of substance	n	mole	mol
luminous intensity	I_v	candela	cd

Table 1: SI Base Units

Other than suggesting the use of T for temperature the SI does not provide guidance on radiometric symbols. What about L and refractive index - see SI brochure.

1.1.2 International Organization for Standardization

The International Organization for Standardization has published a standard for light and related electromagnetic radiations (*ISO*, 1992). Reference to this work will use the abbreviation **ISO92**.

1.1.3 International Union of Pure and Applied Physics

The International Union of Pure and Applied Physics suggests a range of symbols for a number of radiometric quantities in what is colloquially called 'The Red Book' (*Cohen and Giacomo*, 1987). Reference to this work will use the abbreviation IUPAP87.

1.1.4 International Union of Pure and Applied Chemistry

The International Union of Pure and Applied Chemistry have further suggestions on radiometric quantities in what is colloquially called 'The Green Book' (*Cvitaš*, 2007). Reference to this work will use the abbreviation IUPAC07.

1.1.5 A Jury of Twelve

Common usage does not always agree with recommended standards. To demonstrate this a jury of twelve publications have been used to determine which symbols are currently used. These are:

- *Houghton* (1979) (referenced as H79),
- *Goody and Yung* (1989) (referenced as G89),
- *Wayne* (1991) (referenced as W91),
- *Stephens* (1994) (referenced as S94),

- *Salby* (1996) (referenced as S96),
- *Wolfe* (1998) (referenced as W98),
- *Andrews* (2000) (referenced as A00),
- *Liou* (2002) (referenced as L02),
- *Martin* (2004) (referenced as M04),
- *Petty* (2004) (referenced as P04),
- *Applied Spectroscopy* (2007) (referenced as AS07),
- *Zdunkowski et al.* (2007) (referenced as Z07).

Note that not all quantities are used in every text and occasionally a text uses two symbols for the same quantity.

2 Symbols for mathematical variables and physical constants

2.1 Spherical polar notation and solid angle

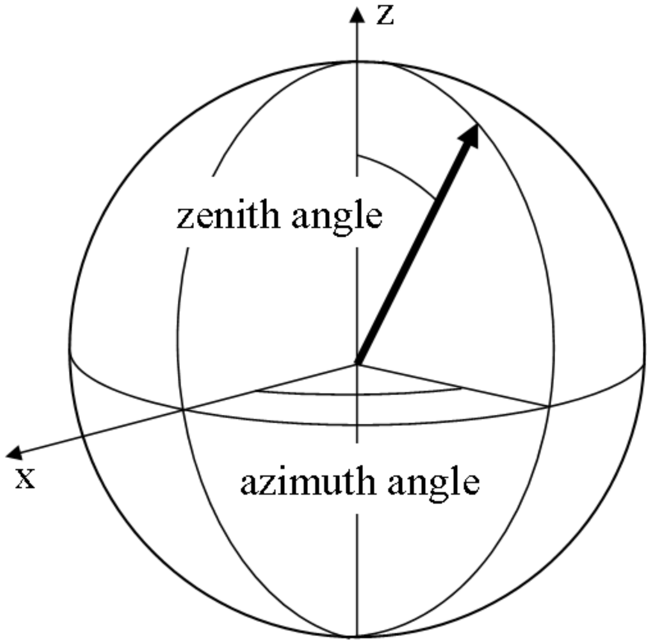

Figure 1: Representation of azimuth and zenith angles.

The azimuth and zenith angles for spherical polar coordinates are defined in Figure 1. Table 2 gives the symbols used by the selected texts. The following symbols are the most widely used and are adopted:

- Azimuth angle, ϕ,
- Zenith angle, θ.

Variable	**Symbol**	**Reference**
Azimuth angle	ζ	G89
	ϕ	S94, S96, W98, L02, M04, P04
	φ	Z07
Zenith angle	ϕ	A00
	θ	H79, W91, S94, S96, W98, L02, M04, P04
	ϑ	Z07

Table 2: Symbols used to represent azimuth and zenith angles.

The solid angle is that fraction of the surface of a sphere that a particular object covers, as seen by an observer at the sphere's centre. Table 3 shows two symbols that are typically used to represent solid angle. No distinction is made between solid angle and solid angle projected onto the reference plane. To introduce

Variable	**Symbol**	**Reference**
Solid angle	Ω	S94, S96, W98, A00, L02, M04, Z07
	ω	H79, G89, P04

Table 3: Symbols used to solid angle.

this distinction the following symbols are adopted:

- Solid angle, ω,
- Projected solid angle, Ω.

2.2 Abbreviation of Spherical and Hemispherical Integration

Using symbols for solid angle or projected solid angle an abbreviated notation can be used to cover integration over solid angle:

upper hemisphere integration

$$\int_0^{2\pi}\int_0^{\frac{\pi}{2}} \sin(\theta)\ d\theta\, d\phi \quad = \quad \int_0^{2\pi} d\omega, \tag{1}$$

lower hemisphere integration

$$\int_0^{2\pi}\int_{\frac{\pi}{2}}^{\pi} \sin(\theta)\ d\theta\, d\phi \quad = \quad \int_0^{-2\pi} d\omega, \tag{2}$$

full hemisphere integration

$$\int_0^{2\pi}\int_0^{\pi} \sin(\theta)\ d\theta\, d\phi \quad = \quad \int_0^{4\pi} d\omega, \tag{3}$$

where $d\omega$ is the differential solid angle which, if capitalised, indicates that a factor $\cos(\theta)$ is included within the integral, e.g.

upper hemisphere integration

$$\int_0^{2\pi}\int_0^{\frac{\pi}{2}} \cos(\theta)\sin(\theta)\ d\theta\, d\phi \quad = \quad \int_0^{2\pi} d\Omega, \tag{4}$$

lower hemisphere integration

$$\int_0^{2\pi}\int_{\frac{\pi}{2}}^{\pi} \cos(\theta)\sin(\theta)\ d\theta\, d\phi \quad = \quad \int_0^{-2\pi} d\Omega, \tag{5}$$

full hemisphere integration

$$\int_0^{2\pi}\int_0^{\pi} \cos(\theta)\sin(\theta)\ d\theta\, d\phi \quad = \quad \int_0^{4\pi} d\Omega, \tag{6}$$

so that $d\Omega = \cos(\theta)\, d\omega$ is the projection of solid angle onto the x-y plane.

2.3 Constants

The symbols for most constants are well established and are reproduced here for reference.

Symbol	**Constant**	**Value**	Reference
k_B	Boltzmann's constant	1.38062×10^{-23} J K^{-1}	
N_A	Avogadro's number	6.02297×10^{23} molecule mol^{-1}	
G	Gravitational constant	6.673×10^{-11} N m^2 kg^{-2}	
h	Planck's constant	6.62620×10^{-34} J sec	
σ	Stefan-Boltzmann constant	5.66961×10^{-8} J m^{-2} s^{-1} K^{-4}	
R	Universal gas constant	8.31432 J mol^{-1} K^{-1}	
c	Velocity of light	2.99792458×10^8 m s^{-1}	

Table 4: Universal constants.

Symbol	**Constant**	**Value**	Reference
g	Acceleration of gravity (at sea level and 45° latitude)	9.80616 m s^{-2}	
ω_e	Angular velocity of rotation of the Earth	7.27221×10^{-5} rad s^{-1}	
m_e	Mass of the earth	5.988×10^{24} kg	
r_e	Mean distance between the Earth and the Sun	1.49598×10^{11} m	
a_e	Mean radius of the Earth	6.37120×10^6 m	
a_s	Mean radius of the Sun (visible disk)	6.96000×10^8 m	
M_a	Molecular weight of dry air	28.97 g mol^{-1}	
E	Solar constant at r_e	1366 W m^{-2}	

Table 5: Physical constants.

3 Symbols used in atmospheric radiative transfer

3.1 Symbols for the spectral description of light

The symbol λ is universally adopted for wavelength. However this is not true for other quantities such as wavenumber or frequency. Table 6 lists symbols used for the spectral description of light. The following

Quantity	Definition	Symbol	Authors
wavelength	distance between points on a wave at the same phase	λ	H79, IUPAP87, G89, W91, **ISO92**, P04, S94, S96, W98, A00, L02, P04, AS07, IUPAC07, Z07
frequency	number of cycles in unit time	f	IUPAP87, **ISO92**, M04, AS07
		ν	H79, IUPAP87, G89, W91, **ISO92**, S94, S96, W98, A00, P04, AS07, IUPAC07, Z07
		$\tilde{\nu}$	L02
wavenumber	number of waves in a unit length	σ	IUPAP87, **ISO92**, W98, IUPAC07 (in a medium), AS07
		$\tilde{\nu}$	H79, S94, P04, AS07, IUPAC07 (in a vacuum?? round wrong way??), Z07
		ν	L02
angular frequency	$2\pi\times$ frequency	ω	**ISO92**
angular wavenumber	$\frac{2\pi}{\lambda}$	k	**ISO92**

Table 6: Symbols for spectral description of light.

symbols are adopted:

- wavelength, λ,
- frequency, ν,
- wavenumber, $\tilde{\nu}$,
- angular frequency, ω,
- angular wavenumber, k.

If it is necessary to differentiate values in a vacuum from those in a medium then a vacuum can be denoted by adding a subscript '0' to the symbol.

3.2 Radiometric Quantities

The transfer of light in the atmosphere is usually expressed in radiometric quantities whose symbols have been slowly evolving. Table 7 gives the **ISO92** standard symbols.

Quantity	**Symbol**		**Definition**	**S.I. Units**
Radiant energy	Q or W		Energy emitted, transferred or received as radiation.	J
Radiant power or radiant energy flux	Φ or P	$= \frac{dQ}{dt}$	Power emitted, transferred or received as radiation,	W
Radiant exitance	M	$= \frac{d\Phi}{dA}$	At a point on a surface the radiant energy flux leaving an element of the surface divided by the area of that element.	$\mathrm{W\,m^{-2}}$
Irradiance	E	$= \frac{d\Phi}{dA}$	At a point on a surface the radiant energy flux incident on an element of the surface divided by the area of that element.	$\mathrm{W\,m^{-2}}$
Radiant intensity	I	$= \frac{d\Phi}{d\omega}$	In a given direction from a source, the radiant energy flux leaving the source, or an element of the source, in an element of solid angle containing the given direction, divided by that element of solid angle.	$\mathrm{W\,sr^{-1}}$
Radiance	L	$= \frac{d\Phi}{d\omega dA\cos\theta}$	At a point on a surface and in a given direction, the radiant intensity of an element of surface, divided by the area of orthogonal projection of this element on a plane perpendicular to the given direction.	$\mathrm{W\,m^{-2}\,sr^{-1}}$

Table 7: Radiometric quantities defined in **ISO92**. The French led the naming and development of radiometric standards so symbols for the last three quantities are from *eclairage* (E), *intensité* (I) and *luminosité* (L).

The use of flux to denote per unit time and density to denote per unit area is somewhat at odds with the more common usage of flux as a per unit area e.g. magnetic flux. Hence radiant power is preferred to radiant energy flux and irradiance is preferred to radiant energy flux density. Secondly the division of irradiance into radiant exitance when it is emitted (scattered or reflected) from a surface or irradiance when it is incident on a surface seems unnecessary. One could equally introduce further nomenclature to describe radiation incident upon, or leaving, a surface for other radiometric quantities. Hence the **ISO92** symbols have been adopted with the exception of radiant exitance. Irradiance will be used to refer to incident and leaving radiant power. The revised list is:

- Radiant energy, Q,
- Radiant power, Φ,
- Irradiance (leaving or incident upon a surface), E,
- Radiant intensity, I,
- Radiance, L.

As discussed in **ISO92** the adjective 'spectral' applied to radiometric quantities can mean one of two things:

- The subscript λ (or ν or $\tilde{\nu}$) is used to indicate that the quantity has the dimension of a derivative with respect to λ (or ν or $\tilde{\nu}$ respectively). The name of a quantity which is a spectral concentration may be shortened by replacing the words *spectral concentration of* by the adjective *spectral*, for example spectral concentration of radiant energy density may be called spectral radiant energy density.

- The adjective *spectral* is also used to designate quantities which are functions of wavelength but which are not spectral concentrations, for example *spectral emissivity*. The functional dependence is usually indicated by including λ as part of the symbol, for example $\epsilon(\lambda)$.

3.3 Transmittance, Optical Path and Optical Depth

Perhaps the least standard are the symbols adopted for transmittance, optical path and optical depth. In addition the words *transmission*, *transmissivity* and *transmittance* are not clearly defined.[1] For example there is no consesnus between the following 'reference' definitions:

transmittance

- The ratio of the transmitted luminous flux to the incident luminous flux. (*Simpson and Weiner*, 1989)
- The ratio of the radiant power transmitted by the sample to the radiant power incident on the sample. (AS07)
- The ratio of the energy transmitted by a body to that incident upon it. (*Walker*, 2007)
- The radiant power transmitted by a body divided by the total radiant power incident upon the body. (*Parker*, 1989)
- The ratio of intensity or radiant power transmitted by a sample to the intensity or radiant power incident upon it. (*Considine*, 2002)
- The ratio of the intensity of a transmitted wave to the intensity of the incident wave. (*Clugston*, 1998)
- The ratio of the energy of some form of radiation transmitted though a surface to the energy falling on it. (*Isaacs*, 1996)

transmission

- Conveyance or passage through a medium, as of light, heat, sound, etc. (*Simpson and Weiner*, 1989)
- Synonym for transmittance. (*Walker*, 2007)
- Synonym for transmittance. (*Parker*, 1989)

transmissivity

- The degree to which a medium allows electromagnetic radiation to pass through it. (*Simpson and Weiner*, 1989)
- The ratio of the transmitted radiation to the radiation arriving perpendicular to the boundary between two mediums. (*Parker*, 1989)

Perhaps the most useful definition is this last one which is in accord with S94 which defines the reflectivity and transmissivity as coefficients for the energy reflected and transmitted at an interface rather than as coefficients that relate to the reflection and transmission of wave amplitudes. Another interesting definition is that of *van der Hulst* (1980) who defines the transmission as the sum of the unattenuated direct beam

[1] Transmittivity is not widely used and will be ignored. It is defined by *Walker* (2007) as the transmittance of unit thickness of a non-scattering medium.

and the forward scattered (or emitted) beam. This underlines the tacit assumption in the definition of transmittance that there are no internal sources within the medium, i.e. the radiant power exponentially decays with distance into the medium. The following definitions are suggested:

transmittance The ratio of the radiant power transmitted by a medium to the radiant power incident on the medium in the absence of internal sources,

transmission A synonym for transmittance,

transmissivity The coefficients for the energy transmitted at an interface.

When internal sources exist the transmission or transmittance can adopt values greater than or equal to one.

Transmittance or transmission can be calculated at a specific frequency or averaged over a range of frequencies. The former term is called the *spectral transmittance* by **ISO92** which also introduces the synonym *spectral transmission factor*. When a spectral average quantity is described the word *spectral* can be replaced by the word *band*.

3.3.1 Transmittance

The **ISO92** recommended symbol for transmittance τ is only notable because of its lack of adoption. Table 8 lists symbols used for spectral transmittance and band transmittance.

Quantity	Symbol	Authors
transmittance	t	M04, P06
	τ	ISO98, W98
	$\tau_{\tilde{\nu}}$	H79
	T_ν	G89, L02
	$\mathcal{T}_\nu$	S94, A00, Z07
	$\mathscr{T}_\lambda$	S96
band transmittance	$\bar{\tau}$	H79
	$\bar{T}_i$	G89
	$T_{\bar{\nu}}$	L02
	$\overline{\mathcal{T}}_{\mathrm{r}}$	A00
	$\mathcal{T}_{\Delta\nu}$	S94
	$\mathcal{T}_{\bar{\nu}}$	Z07
	$\mathcal{T}_i$	P06

Table 8: Symbols for transmittance in common usage. Note that the quantities follow the mathematical definition provided in the individual text irrespective of the terminology used by the text.

The following symbols could be adopted:

- transmittance, $\mathcal{T}$,
- band transmittance, $\overline{\mathcal{T}}$.

To indicate spectral averages an overline could be added for this and other symbols. An alternative approach of using, for instance, a subscript $\Delta\nu$ to indicate a spectral average could be confusing as in other cases this notation indicates a derivative function.

3.3.2 Optical Path and Optical Depth

The terms *optical path*, *optical thickness* and *optical depth* are used very loosely so that in any work one needs to refer to the author's definition. An approach is to describe the logarithmically scaled transmittance though a medium as the optical path, i.e.

$$\chi = -\ln \mathcal{T}.$$

When the optical path is used as a vertical coordinate measured from the top of a plane parallel atmosphere it is called the optical depth. So for an optical path at an angle θ to normal the optical depth is

$$\tau = \chi \cos\theta.$$

Table 9 lists symbols used for optical path, depth or thickness.

Quantity	**Definition**	**Symbol**	**Authors**
monochromatic optical path or thickness		χ_λ	S96
		χ_ν	A00
		τ	M04, P06
monochromatic optical depth		χ	H79
		χ_ν	A00
		t	G89
		τ	G89, S94, L02, M04, P06, Z07
		τ_λ	S96

Table 9: Symbols for optical depth in current usage. Note that the quantities follow the mathematical definition provided in the individual text irrespective of what terminology the text uses. (NEEDS CHECKING)

In order to differentiate optical path and optical depth the following symbols are suggested:

- optical path, χ,
- optical depth, τ,

As the optical path or depth is a scaling of transmittance it can be spectrally averaged. Very few authors make this distinction. A bar can be given above the relevant symbol in these cases.

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