Prelims: Mechanics: 0

1. (a) A car accelerates uniformly from rest to 80 km per hour in 10 s. How far has the car travelled?

   (b) A stone falls from rest with an acceleration of 9.8 ms^{-2}. How fast is it moving after it has fallen through 2 m?

   (c) A car is travelling at an initial velocity of 6 ms^{-1}. It then accelerates at 3 ms^{-2} over a distance of 20 m. What is its final velocity?

2. The brakes on a car of mass 1000 kg travelling at a speed of 15 ms^{-1} are suddenly applied so that the car skids to a stop in a distance of 30 m. Use energy considerations to determine the magnitude of the total frictional force acting on the tyres, assuming it to be constant throughout the braking process. What is the car's speed after the first 15 m of this skid?

3. The gravitational potential energy for a mass m at a distance $R + h$ from the centre of the earth (where R is the radius of the earth) is $\frac{-GMm}{R+h}$ where G is Newton's gravitational constant and M is the mass of the Earth. If $h << R$, show that this is approximately equal to a constant (independent of h) $+mgh$, where $g = GM/R^2$ [hint: write $R + h = R(1 + h/R)$ and expand $(1 + h/R)^{-1}$ by the binomial theorem].

4. Show that the minimum speed with which a body can be projected from the surface of the Earth to enable it to just escape from the earth's gravity (and reach "infinity" with zero speed) is given by

$$v_{\text{escape}} = \sqrt{2GM/R}$$

   where M, G and R are defined as in the previous question. If a body is projected vertically with speed $\frac{1}{2}v_{\text{escape}}$, how high will it get? (Give the answer in terms of R; neglect air resistance throughout this question.)

5. (a) The position of a particle as a function of time is given by $x(t) = A\sin(\omega t + \phi)$ where A, ω and ϕ are constants. Obtain similar formulae for (a) the velocity, (b) the acceleration of the particle. If $\phi = \pi/6$, find in terms of A the value of x at $t = \pi/6\omega$. What is the value of x (in terms of A) when the acceleration is greatest in magnitude?

   (b) A particle of mass m moves in one dimension under the action of a force given by $-kx$ where x is the displacement of the body at time t, and k is a positive constant. Using "$F = ma$", write down a differential equation for x, and verify that its solution is of the form $x = A\cos(\omega t + \phi)$, where $\omega^2 = k/m$. If the body starts from rest at the point $x = A$ at time $t = 0$, find an expression for x at later times.

6. A rocket is fired vertically upwards with constant acceleration from the Earth's surface. After one minute it reaches a height of 36 km, and the motor is shut off. Neglect air resistance and variation of g with height.

   (a) Calculate the maximum height reached.

   (b) Calculate the total time of flight.

   (c) Draw a $v - t$ diagram for the entire flight.

7. Consider the motion of a object able to move along a line under constant acceleration a. At time $t = 0$, its initial position and velocity are x_0 and v_0 respectively. Starting from Newton's 2nd law show that at a later time t,

$$v = v_0 + at, \qquad x = x_0 + v_0 t + \frac{1}{2}at^2.$$

   Eliminate the time between these two relations to show that

$$a(x - x_0) \quad = \quad \frac{1}{2}(v^2 - v_0^2).$$

   Use this to find the minimum stopping distance for a car travelling at 100 $\mathrm{km\,h^{-1}}$ if its maximum deceleration is $10\,\mathrm{ms^{-2}}$.

8. Estimates of inertia and forces:

   (a) A cabin cruiser of 15 tonnes (1 tonne $= 10^3$ kg) drifts in towards a dock at a speed of 0.3 $\mathrm{ms^{-1}}$ after its engines have been stopped. A woman on the dock is able to reach the boat with a boat hook when it is 2 m from the dock and thereafter can push with a constant force of 400 N. Can she bring the boat to rest before it hits the dock?

   (b) A man of mass 80 kg jumps down on a paved road from a height of 0.5 m. He does not bend his knees on landing so his motion is arrested in about 2 cm. What is the average force on his bone structure?

   (c) If the man now tries jumping from a height of 1.5 m but does bend his knees on impact so that his centre of gravity descends a further distance h after his feet touch the ground, what must h be so that the average force exerted on him by the impact is only five times his normal weight?

9. When a car corners at speed one is relying on the friction between the tyres and the road to counter the centrifugal force tending to throw the car outwards. One way to reduce the dependence on friction is to 'bank' the corner. For a car travelling with a speed v entering a corner of radius r, show that the angle of the bank, θ, (measured from the horizontal) should satisfy $\tan\theta = v^2/(gr)$ if there is to be no reliance on friction. In the light of this result discuss the design of the 'Wall of Death' fairground attraction in which a motorbike is ridden at speed around the inner surface of a large vertical cylinder, with the bike and rider at a small angle to the horizontal plane. What role does friction play and why is a small angle necessary?

10. Calculate the following centripetal accelerations as fractions or multiples of g and comment on the results:

    (a) the acceleration towards the Earth's axis of an object resting on the Earth's surface at 45°latitude. The Earth's radius is 6378 km.

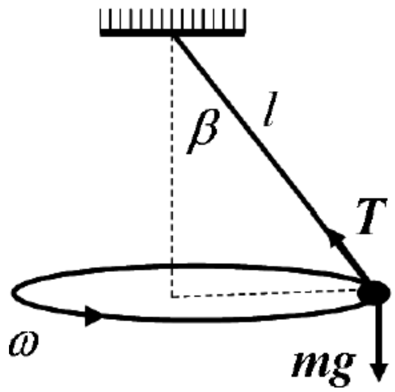
    (b) the acceleration of the Moon towards the Earth. The radius of the Moon's orbit is 384399 km and its period is 27.3 days.

    (c) the acceleration of an electron moving around a proton at a speed of 0.007c where c is the speed of light in an orbit of radius 0.05 nm (the first Bohr orbit of the H atom).

11. The muzzle velocity of a gun is 60 ms^{-1}. A woman shoots one shot each second straight up into the air, which may be considered frictionless. How many bullets will be in the air at any time and at what height above the ground will they pass each other?

12. An object fixed with respect to the surface of a planet identical in mass and radius to the Earth, experiences zero apparent gravity at the equator. What is the length of a day on that planet? What apparent gravity would be experienced by an object at the planet's poles? [Apparent gravity is the force on a stationary object taking into account both gravitational attraction and rotation.]

13. A conical pendulum consists of a mass m suspended by a massless string of length l as shown below. The mass rotates in a horizontal circle at fixed angular velocity ω so that the string makes a constant angle β with the vertical. Show that the angular velocity of rotation is given by $\omega = \sqrt{g/lcos\beta}$.



14. A ladder leans against a frictionless wall while the bottom rests on a horizontal floor with coefficient of friction μ as shown below. Show that the smallest angle that the ladder can make with the floor without slipping is given by $\tan\theta_{\min} = 1/2\mu$.