

Prelims: Mechanics: 0

1. (a) A car accelerates uniformly from rest to 80 km per hour in 10 s. How far has the car travelled?
 (b) A stone falls from rest with an acceleration of 9.8 ms^{-2} . How fast is it moving after it has fallen through 2 m?
 (c) A car is travelling at an initial velocity of 6 ms^{-1} . It then accelerates at 3 ms^{-2} over a distance of 20 m. What is its final velocity?
2. The brakes on a car of mass 1000 kg travelling at a speed of 15 ms^{-1} are suddenly applied so that the car skids to a stop in a distance of 30 m. Use energy considerations to determine the magnitude of the total frictional force acting on the tyres, assuming it to be constant throughout the braking process. What is the car's speed after the first 15 m of this skid?
3. The gravitational potential energy for a mass m at a distance $R + h$ from the centre of the earth (where R is the radius of the earth) is $\frac{-GMm}{R+h}$ where G is Newton's gravitational constant and M is the mass of the Earth. If $h \ll R$, show that this is approximately equal to a constant (independent of h) $+mgh$, where $g = GM/R^2$ [hint: write $R + h = R(1 + h/R)$ and expand $(1 + h/R)^{-1}$ by the binomial theorem].
4. Show that the minimum speed with which a body can be projected from the surface of the Earth to enable it to just escape from the earth's gravity (and reach "infinity" with zero speed) is given by

$$v_{\text{escape}} = \sqrt{2GM/R}$$

where M , G and R are defined as in the previous question. If a body is projected vertically with speed $\frac{1}{2}v_{\text{escape}}$, how high will it get? (Give the answer in terms of R ; neglect air resistance throughout this question.)

5. A rocket is fired vertically upwards with constant acceleration from the Earth's surface. After one minute it reaches a height of 36 km, and the motor is shut off. Neglect air resistance and variation of g with height.
 - (a) Calculate the maximum height reached.
 - (b) Calculate the total time of flight.
 - (c) Draw a $v - t$ diagram for the entire flight.
6. Consider the motion of a object able to move along a line under constant acceleration a . At time $t = 0$, its initial position and velocity are x_0 and v_0 respectively. Starting from Newton's 2nd law show that at a later time t ,

$$v = v_0 + at, \quad x = x_0 + v_0t + \frac{1}{2}at^2.$$

Eliminate the time between these two relations to show that

$$a(x - x_0) = \frac{1}{2}(v^2 - v_0^2).$$

Use this to find the minimum stopping distance for a car travelling at 100 km h^{-1} if its maximum deceleration is 10 ms^{-2} .

7. Calculate the following centripetal accelerations as fractions or multiples of g and comment on the results:
 - (a) the acceleration towards the Earth's axis of an object resting on the Earth's surface at 45° latitude. The Earth's radius is 6378 km.
 - (b) the acceleration of the Moon towards the Earth. The radius of the Moon's orbit is 384399 km and its period is 27.3 days.
 - (c) the acceleration of an electron moving around a proton at a speed of $0.007c$ where c is the speed of light in an orbit of radius 0.05 nm (the first Bohr orbit of the H atom).
8. The muzzle velocity of a gun is 60 ms^{-1} . A woman shoots one shot each second straight up into the air, which may be considered frictionless. How many bullets will be in the air at any time and at what height above the ground will they pass each other?
9. An object fixed with respect to the surface of a planet identical in mass and radius to the Earth, experiences zero apparent gravity at the equator. What is the length of a day on that planet? What apparent gravity would be experienced by an object at the planet's poles? [Apparent gravity is the force on a stationary object taking into account both gravitational attraction and rotation.]
10. A ladder leans against a frictionless wall while the bottom rests on a horizontal floor with coefficient of friction μ as shown below. Show that the smallest angle that the ladder can make with the floor without slipping is given by $\tan \theta_{\min} = 1/2\mu$.

